Research article

MATHEMATICAL MODEL ON E.COLI KINETICS INFLUENCED BY RETARDATION ON DECAY PHASE ADSORPTION AND DESORPTION IN SOIL AND WATER ENVIRONMENT IN PORT HARCOURT, RIVERS STATE OF NIGERIA

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Abstract

The behaviour of E.coli are expressed in several conditions, the kinetics of E.coli are expressed by the formation characteristics in the study location, the rate of retardation are not left behind, it is express in the system, because it has a reflection on the formation variation in the study area.. Retardation factor are considered because of it influences from formation characteristic in the study location, since structural stratification under geological setting played major roles in the system dynamics, desorption and adsorption are determined by the geological setting, the kinetics of E.coli are influence by the deposition of the formation, it is reflected on the deposition of substrate and microbes in soil and water environment, kinetics of E.coli are influence through the stratification variation in soil, these are reflected in the kinetic reaction in the formations, several concept has been express on kinetics of microbes, but the study carried out is precisely on the deposition of E.coli in the study area, this to determine the rate of kinetics influence in the study area, the influences were found to reflect on the retardation on decay phase of E.coli in the formation, this were found on the slow level on rate of migration, there is no perfect method to predict the rate of kinetic influence on retardation through decay phase of E.coli in the study location, base on this condition mathematical model was develop to monitor the kinetic rate of E.coli on retardation in soil and water environment. Adsorption and desorption rate were express in the system, because the rate of retardation which that the microbes may be experiencing degradation. The model were generated through the derived governing equation, the model expressed will be useful to experts to monitor the rate of kinetics influence including retardation on decay phase condition in soil and water environments. Copyright © IJESTR, all rights reserved.

Keywords: mathematical model, kinetics of E.coli, adsorption desorption and water environments.

1. Introduction

To forecast the occurrence of pathogens in water, a disconnected group of microorganisms is usually used, generally known as fecal indicator organisms (Payment et al., 2005, Foppen, 2007). Many microbes have been recommend as microbial indicators of fecal contamination (like enterococci, coliphages and sulphite reducing

clostridial spores; Medema et al., 2003), but two of the most important indicators used worldwide are Escherichia coli and thermotolerant coliform bacteria (for microbiological definitions of these indicators. Both are widely used, because their detection is relatively simple, fast, and reliable. E. coli is the preferred indicator of fecal contamination, as it is the only member of the thermotolerant coliform group that is invariably found in feces of warm-blooded animals and it outnumbers the other thermotolerant coliforms in both human and animal excreta (Medema et al., 2003). Thermotolerant coliforms are a less reliable index of fecal contamination than E. coli, although under most circumstances their concentrations are directly related to E. coli concentrations (Payment et al., 2003). Viruses may be considered as the most critical or limiting microorganism. Because of their small size, their mostly negative surface charge, and their high persistence in the environment, they may travel long distances in the subsurface. In addition, they can be highly infectious (Schijven, 2001). In the study by Karim et al. (2004a), although E. coli and thermotolerant coliforms as representatives of the group of fecal indicator organisms have often been found in groundwater systems, to date there has been no comprehensive report evaluating and discussing their transport characteristics. Some of the reviews concentrate on the movement of bacteria and viruses in aquifers in a qualitative way, without attempting to predict their migration (e.g. Romero, 1970; Lewis et al., 1980; Hagedorn et al., 1981; Crane and Moore, 1984; Bitton and Harvey, 1992; Stevik et al., 2004). Others mainly focus on first-order die-off rates, thereby neglecting the transport component including attachment and detachment processes (e.g. Reddy et al., 1981; Barcina et al., 1997). Murphy and Ginn (2000) mainly summarize the mathematical descriptions of the various physico-chemical and biological processes involved in the transport of bacteria and viruses, without indicating the relative importance of these processes and their occurrence in the natural environment. Merkli (1975) and Althaus et al. (1982) have presented a comprehensive bacteria transport model based on the colloid filtration theory (Herzig et al., 1970; Yao et al., 1971), including the effects of dispersion, diffusion, sedimentation, and filtration. The effects of retardation due to equilibrium adsorption were also included, as well as the physical, chemical, and biological factors determining the die-off of pathogens and indicator organisms. Their findings, however, were based on relatively few data, derived from batch experiments and equilibrium sorption.

2. Theoretical background

Kinetics of E. coli Microbial growth on and exploitation of ecological pollutants as substrates have been studied by many researchers. Most times, substrate exploitation results in removal of chemical contaminant enhance in microbial biomass and succeeding biodegradation of the contaminant. The relation between the specific growth rate (m) of a population of microorganisms and the substrate concentration (S) is a valuable tool in biotechnology. This relationship is represented by a set of several empirically derived rate laws referred to as theoretical models. These models are nothing but mathematical expressions generated to describe the behaviour of a given system, but mathematical equations develop is to apply this concept to monitor the behaviour of E.coli transport in unconfined formation. Several mathematical concepts has been applied on microbial kinetics in different dimensions globally but the adopted concept in the study are directly for E.coli under the family fecal Coliforms, the model will establish the behaviour of the system in retardation phase under the influence of decay condition causing biodegradations in the transport process in soil and water environments the decay phase are under the state of adsorption, this rate substance accumulation of this microbes are found to accumulate more in the organic an lateritic soil, the retardation influence can be attributed to high degree of temperature that may be faviourable to E.coli in the formation especially the unsaturated zone, it can also as result of absent of substrate in those formation. Formation variation played major, thus the phase of desorption since the formation varies in the study area the rate of adsorption are influence by the stratifications variation reflected from the geological setting, the rate of desorption's are base on the formation characteristics deposition in the study area, therefore the degree of the formation characteristics are base on variation expressed the formation. The kinetics of E.coli in the system is base on the behaviour of microbes in the base of the stratifications reflected from the rate of substrate depositions in the formations. Growth is an increase in cellular constituents that may result in an increase in cell size, an increase in cell number, or both. When an experts speaks of microbial growth it is usually increase in cell number that he/she is after. Consequently, there is a tendency for experts to follow microbial growth as populations rather than following the growth of individual cells. Experts tend to be more interested in population sizes than the size (mass) of any individual cell. Typical measurement of microbial growth will be done over the span of more than one microbial generation. An increase in cell number is an immediate consequence of cell division. Increase in cell numbers occurs when microorganisms reproduce by a process like budding or binary fission. Budding is a form of reproduction in which a new cell is formed as an outgrowth from the parent cell, as in the case of yeast and some bacteria. The majority of bacteria reproduce by a mechanism termed binary fission. The basic hypothesis of biodegradation kinetics is that substrates are consumed via catalyzed reactions carried out only by the organisms with the obligatory enzymes. Therefore, rates of substrate degradation are generally proportional to the catalyst concentration (concentration

of organisms able to degrade the substrate) and dependent on substrate concentration characteristic of diffusion kinetics (e.g. Michaelis-Menten and Monod kinetics). Saturation kinetics suggests that at low substrate concentrations (relative to the half-saturation constant), rates are approximately proportional to substrate concentration (first order in substrate concentration), while at high substrate concentrations, rates are independent of substrate concentration (zeroorder in substrate concentration). In the case of substrates that contribute to the growth of the organisms, rates of substrate degradation are linked to rates of growth (i.e. the concentration of the biomass increases with substrate depletion). The basic hypothesis of biodegradation kinetics is that substrates are consumed via catalyzed reactions carried out only by the organisms with the requisite enzymes (Okpokwasili and Nweke, 2005. Therefore, rates of substrate degradation are generally proportional to the catalyst concentration (concentration of organisms able to degrade the substrate) and dependent on substrate concentration characteristic of saturation kinetics (e.g. Michaelis-Menten and Monod kinetics). Saturation kinetics suggests that at low substrate concentrations (relative to the half-saturation constant), rates are approximately proportional to substrate concentration (first order in substrate concentration), while at high substrate concentrations, rates are independent of substrate concentration (zeroorder in substrate concentration). In the case of substrates that contribute to the growth of the organisms, rates of substrate degradation are linked to rates of growth (i.e. the concentration of the biomass increases with substrate depletion) (Okpokwasili and Nweke, 2005).

3. Governing Equation

$$N\frac{\partial ho}{\partial t} = hs\frac{\partial h^f o}{\partial X} - VKy\frac{\partial h^f o}{\partial X} \qquad (1)$$

The governing equation are developed in different dimensions from several method by other researchers, because there are other conditions considered in the study as present in the governing equation, the rate desorption and adsorption were introduce in the studies, because the degradation of microbes are influence by the formation characteristic that may influence the microbe E.coli to decrease or increase in soil and water environments. Such condition has been streamline in the study it is correlated in the study to evaluate the behaviour of E.coli in the soil and water, its kinetics is reflected in the growth rate and the biomass of the microbe E.coli reflecting on the substrate, The mathematical analysis of such growth-linked systems is more complex than those situations where growth can be ignored. There are a number of situations where it may not be possible to quantify the concentration of substrate-degrading organisms in a heterogeneous microbial community. However, the rate of substrate depletion can be measured. There are also situations in which the organism concentration remains essentially constant even as the substrate is degraded (i.e. no growth situation). Given these various features of biodegradation kinetics, different models including first-order, zero-order, logistic, Monod (with and without growth) and logarithmic models can be used to describe biodegradation. Applying physical splitting techniques

$N\frac{\partial h^f o}{\partial t} = hs\frac{\partial h^f o}{\partial X}$	 (2)
$\begin{array}{c} x = 0 \\ t = 0 \end{array}$	 (3)
$ \begin{array}{c} h_{(o)} = h_o \\ \partial ho_1 \mid = 0 \end{array} $	
$\overline{\partial t} \mid t = 0$ $N \frac{\partial h o_2}{\partial t} = -V K y \frac{\partial h^f o_2}{\partial t}$	(4)
$\begin{array}{c} x \\ \partial t \\ x = 0 \\ t \\ \partial t \end{array} \right) \partial X$	(.)
$ \begin{array}{c} t = 0 \\ h_{(o)} = h_o \\ \end{array} $	 (5)
$\frac{\partial ho_2}{\partial t} \begin{vmatrix} = 0 \\ t = 0, B \end{vmatrix}$	

This expressed equation are splited according to the various condition that influence the microbial kinetics, several condition were considered in the system the rate of kinetics influence on retardation condition were descretize according to the state of retardation, this also include the condition found on the state of adsorption and desorption state in the formation, several factors are considered in the system and the expressed equation are developed in according to various state of the system, further condition on microbial behaviour E.coli were also considered in the expressed equation, the equation are splited according to these conditions in the system. Apply direct migration on (2)

$$N\frac{\partial h^{f}o}{\partial t} = Nhs + Ki \tag{8}$$

Again, integrate equation (8) directly yields

$Nhs + Nhst + Kt + K_2$	(9)
$hsh_o = K_2$	(10)
And subjecting equation (8) to (3)	
$0 = hsh_o + K_2$	(11)
So that, we put (10) and (11) into (9), we have	
$Nhs = h_o hst - hsh_o t + Nh_o$	(12)
$Nh_{o1} = hsh_ot = Nh_o - hsh_ot$	
$\Rightarrow h_{o1} (N-hst) = h_o (N-hst)$	
$\Rightarrow h_{a1} = h_{a2}$	(14)

Hence, equation (14) entails that at any given distance x, we have constant concentration of the contaminants in the system

The expression in equation [14] shows the rate of Constant concentration of the microbes, therefore the state and rate of microbial kinetics E.coli were found to express constant kinetic functions, this can be attributed to the deposition of the substrate under the influence of kinetics, this condition were expressed on the rate of kinetics influence in retardation phase of the system.

$$\frac{Nh^{f}o}{\partial t} = -VKy\frac{\partial h^{f}o}{\partial X} \qquad \dots \dots \qquad (4)$$

We approach the system by using the Bernoulli's method of separation of variable

$$h^f o = XT \tag{15}$$

i.e.
$$\frac{\partial h^f o_2}{\partial t} = XT^1$$
(16)

$$\frac{\partial ho_2}{\partial X} = X^1 T \tag{17}$$

Put (16) and (17) into (15), so that we have

$$\frac{NT^1}{T} = VKy \frac{X^1}{X} = -\lambda^2 \tag{19}$$

Hence

$$\frac{NT^1}{T} + \lambda^2 = 0 \tag{20}$$

$$X^1 + \lambda^2 X = 0 \tag{21}$$

i.e.

From (21) $T = ACos \frac{\lambda}{\sqrt{N}}t + B Sin \frac{\lambda}{\sqrt{V}Ky}x$

And (16) gives

(23)

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The movement of E.coli in the soil formation are influenced several conditions, the migration of the microbes from one formation to another are determine from the rate void ratio, the degree of soil void influences the rate migration, the flow path in the soil definitely determine other influence like dispersion coefficients in the formation, the established model in [24] express the rate of microbial migration under the influence of distance at various strata, the kinetic of E.coli continue to vary under the influence change in depth, because the variations in formation characteristics determine the behaviour of the microbes. The level of E.coli growth rate can be expressed through the rate of deposition of substrate at different formation; these express the variation of E.coli kinetics in the system.

By substituting (23) and (24) into (15) we get

Subject equation (25) to conditions in (5), so that we have

$$h^f o = Ac \tag{26}$$

: Equation (26) becomes

Again at
$$\frac{\partial ho_2}{\partial t} \bigg| = 0, \ x = 0$$

 $t = 0, B$

Equation (27) becomes

Which is the substrate utilization for microbial growth (population), so that

Substrate utilization is the determinants of microbial growth rate, the rate of increase are through the deposition of the [NKP] in the formations, microbial kinetic of E.coli are expressed base on the of influences found in the

strata in the transport system, such condition can develop several variation of E.coli in the formation, more so Growth is an increase in cellular constituents that may result in an increase in cell size, an increase in cell number, or both. In this condition experts should speaks of microbial growth, it is usually increase in cell number that he/she is after An increase in cell number is an immediate consequence of cell division. Increase in cell numbers occurs when microorganisms reproduce by a process like budding or binary fission. Budding is a form of reproduction in which a new cell is formed as an outgrowth from the parent cell, as in the case of yeast and some bacteria. The majority of bacteria reproduce by a mechanism termed binary fission.

So that equation (27) becomes

$$h^{f}o = ho \ell^{\frac{-n^{2}\pi^{2}N}{2VK_{y}}x} \cos \frac{n\pi}{2}t \qquad(33)$$

Now, we consider equation (6) which is the steady flow state of the system

The formation of the soil has a serious influence in this condition, this is base on the stratification of the formation uniform flow are base in the strata deposition, the expression in [33] shows the rate of steady state of E.coli deposition in soil under the influence of homogeneous stratification in the formation, such structural deposition develop homogeneous void in the lithology including the flow path under the influences of tortuosity in the study area, these influence the steady state of the microbes and the kinetics states of the microbes are influenced by this expression. More so the rate of retardation come in to play because the formation at steady state develop the retarding state of the formation, this condition develop biodegrading other formation influence are experienced in the steady state phase.

$$hs \frac{\partial h^f o}{\partial X} = -VKy \frac{\partial h^f o}{\partial X} \qquad \dots \dots \qquad (6)$$

Using Bernoulli's method, we have

 $h^f o_3 = XT \tag{34}$

$$\frac{h^f o_3}{\partial X} = XT^1 \tag{35}$$

$$\frac{h^{\prime} o_{3}}{\partial X} = X^{1}T$$
(36)

Put (35) and (36) into (6), so that

i.e.
$$hs \frac{X^{1}}{X} = -VKy \frac{X1}{X} = \varphi$$
(38)
 $hs \frac{X^{1}}{X} = \varphi$ (39)

$$-VKy\frac{X^{1}}{X} = \varphi \tag{40}$$

$$\therefore X = A \ell^{\frac{\varphi}{hs}x}$$
 (41)

And $X = B \ell^{\frac{\psi}{VKy^x}}$ (42)

Put (41) and (42) into (34), gives

Subject equation (44) to (7) yields

$$ho_3 = (0) = h^{\dagger} o$$
(44)

So that equation (45) becomes

$$h^{f}o_{3} = h^{f}o\ell^{(x-x)\frac{\phi}{VKy}}$$
(45)

Now, assuming that at the steady flow there is no NKP for substrate utilization, our concentration is zero so that equation (46) becomes

$$ho_3 = 0$$
(46)

The expression in [45] consider the condition when there no substrate in the formation of the soil, the microbes will take the next step of applying adaptation in those formations, the microbes may also experience biodegradation this condition determine the kinetics state of the microbes, the rate of retardation are also experienced in the conditions, this implies that the microbes will become slow in activities including the biodegradation of the microbes in the formations, biodegradation is the decay phase of the formation, its characteristics determine the behaviour of the microbes rate of kinetics in the system. The expression in 46 implies that the concentration will become zero in those formations. Therefore, solution of the system is of the form

$$h^{f}o = h^{f}o_{1} + h^{f}o_{2} + h^{f}o_{3}$$
(47)
We now substitute (14), (33) and (47) into (48), so that we have the model of the form

We now substitute (14), (33) and (47) into (48), so that we have the model of the form $\frac{2}{3} - 2M$

$$h^{f}o = h^{f}o + h^{f}o \ \ell \frac{-n \ \pi \ N}{2VKy} t \ \cos \frac{n\pi}{2} x \qquad(48)$$

$$\Rightarrow \left[h^f o = h^f o \left(1 + \ell \frac{-n^2 \pi^2 N}{2V K y} Cos \frac{n \pi}{2} x \right) \right]$$
(49)

The expression in [49] is the final mathematical model that will monitor the kinetics of E .coli and retardation in decay phase in the study area. The rate retardation were expressed under the influence of decay phase condition, several influence has been stressed in the study, they developed various conditions in the system, the rate of formation characteristic remain the major variables that determine the behaviour of the microbes in soil and water environments. The rate kinetic variation are express on the variations of the formations, structural

strata deposition has been confirmed to develop extensive influence on the kinetics development of E.coli in the study location, but the rate of retardation factors are base on the formation, mostly depend on the rate of substrate and the temperature of the soil. This condition defined the decay phase of the E.coli in the formations; such expression has been defined in the rate of microbial adaptation under the variation of temperature. The soil structural deposition express the rate of desorption and adsorption of the microbes, in most condition the stratification of the formation determined rate accumulations of substance and degradation express the de rate desorption of the substance , the condition were considered in the study, the rate formation characteristic generated the rate of adsorption and desorption of the substances , this also play some roles in the retardation of the microbes, it is reflected from the rate of microburtients deposition in soil and water environments.

4. Conclusion

The behaviour of E.coli kinetics has been evaluated, kinetics is determined on the deposition of the substrate in the system. The rate of E.coli kinetics are express mathematically through the parameters established in the formations, such developed generated the governing equation, the expressions were generated through the variables, the governing equation were splited according to the behaviour of the microbes including the formations variables. The deposition of substrate deposition in the formation developed lots of variation in stratifications, the variation of substrate affect the growth rate and transport process of E.coli in the study location. Formation variables directly establish a relationship with the variation of substrate deposition thus lithology of the formation in the study area. The expressions are reflected on the kinetics state of the microbes under the influence of formation characteristics in the study area. The rate retardation are reflected on the biodegradations of the microbes under the influences of substrate reduction, these are were the growth rate of the microbes are reduced in there microbial population, the condition of the microbes depend on the formation characteristics thus the rate of substrate in the study area, the kinetics reaction depends on these condition state in the study area.

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